38$^{\text{th}}$ IrMO – Round 1

Problem 1

A father and son are chatting when the son poses this question: "Dad, I'm 22 now, but just how old are you?" The father replies, "You wanna know how old your old man is, eh? Hmm, well, I tell you what. I'm as old as your age, plus half of my age." How old is the father?

Problem 2

Erin builds a $10 \times 10 \times 10$ cube out of 1000 identical smaller cubes. She then paints all the faces of the finished cube red. How many of the smaller cubes will end up with exactly two red faces?

Problem 3

Find a two digit number AB which is equal to $(A + B)^2$.

Problem 4

A journalist interviewed participants of the World's Biggest Liar competition. The name of the winner was mentioned to her. She recorded the following statements, of which exactly one is true:

- Aoife is the winner.
- Neither Conor nor Emily won the competition.
- Brian is the winner.
- The winner is either Aoife, Donal or Emily.
- Donal is the winner.

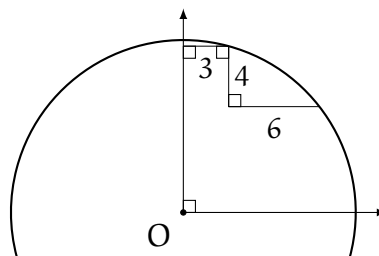Who is the winner of the competition?

Problem 5

In the town of Hibernia, there is a single street which runs in a straight line with houses on one side only. Jenny has been given the task of visiting the houses one by one from left to right, painting numbers on the houses starting with the number 1. However, the citizens consider multiples of 7 to be unlucky, and Jenny is told that she must skip any number that is a multiple of 7. Thus she must paint the houses in order as $1, 2, 3, 4, 5, 6, 8, 9$, and so on.

What number does Jenny paint on the 100th house she visits?

Problem 6

The centre of the circle in the diagram below is the point O. The radius of the circle is r.

Find r^2, the square of the radius of this circle.

Time allowed: 30 minutes